

# Astrophysical Neutrino Reactions and Muon Capture in Deuterium\*

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## Abstract

We discuss the importance of precise study of muon capture in deuterium for correct understanding of some fundamental astrophysical processes.

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## I. WHY THE SUN SHINES

The weak nuclear interaction plays crucial role in the formation of stars in our Universe: it starts the pp chain in stars of the size of our Sun [1, 2], which is the source of its energy. This chain has three branches, ppI, ppII and ppIII [3]. The ppI chain is

$$p + p \rightarrow d + e^+ + \nu_e, \quad (1.1)$$

$$p + p + e^- \rightarrow d + \nu_e, \quad (1.2)$$

$$d + p \rightarrow {}^3\text{He} + \gamma, \quad (1.3)$$

$${}^3\text{He} + {}^3\text{He} \rightarrow {}^4\text{He} + 2p. \quad (1.4)$$

In its turn, the chain ppII is

$${}^3\text{He} + {}^4\text{He} \rightarrow {}^7\text{Be} + \gamma, \quad (1.5)$$

$${}^7\text{Be} + e^- \rightarrow {}^7\text{Li} + \nu_e, \quad (1.6)$$

$${}^7\text{Li} + p \rightarrow 2{}^4\text{He}, \quad (1.7)$$

whereas the ppIII chain is

$${}^7\text{Be} + p \rightarrow {}^8\text{B} + \gamma, \quad (1.8)$$

$${}^8\text{B} \rightarrow {}^8\text{Be}^* + e^+ + \nu_e, \quad (1.9)$$

$${}^8\text{Be} \rightarrow 2{}^4\text{He}. \quad (1.10)$$

Besides, the so called hep reaction takes place

$$p + {}^3\text{He} \rightarrow {}^4\text{He} + e^+ + \nu_e. \quad (1.11)$$

In these chains, the following reactions occur, triggered by the weak nuclear interaction,

$$p + p \rightarrow d + e^+ + \nu_e, \quad (1.12)$$

$$p + p + e^- \rightarrow d + \nu_e, \quad (1.13)$$

$$p + {}^3\text{He} \rightarrow {}^4\text{He} + e^+ + \nu_e, \quad (1.14)$$

$${}^7\text{Be} + e^- \rightarrow {}^7\text{Li} + \nu_e, \quad (1.15)$$

$${}^8\text{B} \rightarrow {}^8\text{Be}^* + e^+ + \nu_e. \quad (1.16)$$

The total neutrino flux from the Sun at the surface of Earth is  $\approx 6.4 \times 10^{10}/\text{cm}^2 \text{ s}$ . The neutrinos produced in the reaction (1.16) have a continuous spectrum with the maximum energy 15 MeV. They have recently been registered in the SNO detector [4–6] via the reactions

$$\nu_x + d \rightarrow \nu'_x + n + p, \quad (1.17)$$

$$\nu_e + d \rightarrow e^- + p + p, \quad (1.18)$$

induced by the weak nuclear interaction, too. The measured total flux of active-flavor neutrinos is  $\approx 5 \times 10^6/\text{cm}^2\text{s}$ , whereas the flux of electron neutrinos is  $\approx 1.7 \times 10^6/\text{cm}^2\text{s}$ . The neutral current to charged current ratio [5] established unambiguously the presence of an active neutrino flavor other than  $\nu_e$  in the observed solar neutrino flux, thus confirming definitely the phenomenon of the neutrino oscillations and that the neutrinos possess a finite mass. When the data from all solar neutrino experiments is combined with the KamLAND data [7], one obtains  $\theta_{12}=34.4^{+1.3}_{-1.2}$  degrees and  $\Delta m_{12}^2=7.59^{+0.19}_{-0.21} \times 10^{-5} \text{ eV}^2$  [8].

## II. WEAK REACTIONS IN LABORATORY

However, the reactions (1.12)-(1.18) cannot be studied experimentally with the desired accuracy in terrestrial conditions at present. In order to grasp them, one should address other weak processes in few-nucleon systems that are feasible in laboratories, such as

$$n \rightarrow p + e^- + \nu_e, \quad (2.1)$$

$$\mu^- + p \rightarrow n + \nu_\mu, \quad (2.2)$$

$$\mu^- + d \rightarrow n + n + \nu_\mu. \quad (2.3)$$

$${}^3H \rightarrow {}^3He + e^- + \bar{\nu}_e, \quad (2.4)$$

$$\mu^- + {}^3He \rightarrow {}^3H + \nu_\mu, \quad (2.5)$$

The one-nucleon weak reactions (2.1) and (2.2) are now experimentally and theoretically well explored. The neutron lifetime is  $\langle \tau \rangle_{world av.} = 879.9 \pm 0.9$  s [9] and the singlet capture rate for the reaction (2.2),  $\Lambda_s = 725.0 \pm 17.4$  s<sup>-1</sup>, has been measured by the MuCap Collaboration at PSI [10]. As to reactions (2.4) and  ${}^3He(\mu^-, \nu_\mu){}^3H$  (2.5), they have been studied in great detail as well. As a result, the half-life of the triton is known with an accuracy  $\sim 0.3$  %,  $(fT_{1/2})_t = (1129.6 \pm 3)$  s [11], and the capture rate of the reaction  ${}^3He(\mu^-, \nu_\mu){}^3H$ ,  $\Lambda_0 = 1496 \pm 4$  s<sup>-1</sup> [12, 13] is also known with the same accuracy. The situation with the reaction  ${}^2H(\mu^-, \nu_\mu)nn$  (2.3) is less favorable so far. Indeed, the last measurements of the doublet capture rate provided  $\Lambda_{1/2} = 470 \pm 29$  s<sup>-1</sup> [14] and  $\Lambda_{1/2} = 409 \pm 40$  s<sup>-1</sup> [15]. As we shall discuss in this talk, the experiment planned by MuSun Collaboration [16], which intends to measure  $\Lambda_{1/2}$  with an accuracy of  $\sim 1.5$  %, will help to clarify essentially the situation in the theory of reactions triggered by weak nuclear interaction in few-nucleon systems.

## III. NUCLEAR MATRIX ELEMENT OF THE WEAK INTERACTION

In order to describe these semi-leptonic reactions one should know how to calculate the matrix element of the weak Hamiltonian  $H_W$  between the initial and final nuclear states,  $|i\rangle$  and  $|f\rangle$ , respectively. Such a matrix element enters cross sections and capture rates. One can write generally,

$$\langle f | \hat{H}_W | i \rangle = -\frac{G_W}{\sqrt{2}} \int d\vec{x} e^{i\vec{q}\cdot\vec{x}} [\vec{j}(0) \cdot \vec{J}_W^a(\vec{x})_{fi} - j_0(0) J_{W,0}^a(\vec{x})_{fi}]. \quad (3.1)$$

The weak lepton current  $j_\mu$  is the four-vector, given in the V-A theory of the weak interactions, e.g., for the muon capture

$$j_\mu(0) = i\bar{u}(\vec{\nu}) \gamma_\mu (1 + \gamma_5) u(\vec{k}). \quad (3.2)$$

The Dirac spinor  $u(\vec{k})$  corresponds to the initial muon with the momentum  $\vec{k}$  and the spinor  $\bar{u}(\vec{\nu})$  corresponds to the final muon neutrino with the momentum  $\vec{\nu}$ .

Looking at the matrix element (3.1) we conclude that the problem lies in constructing the hadron currents and the nuclear wave functions. In its turn, the isovector one-nucleon weak current  $J_{W,\mu}^a$  is also of the V-A form,

$$J_{W,\mu}^a(q_1) = J_{V,\mu}^a(q_1) + J_{A,\mu}^a(q_1), \quad (3.3)$$

where the vector part is given by the matrix element of the isovector Lorentz 4-vector current operator between the nucleon states,

$$\hat{J}_{V,\mu}^a(q_1) = i \left( g_V(q_1^2) \gamma_\mu - \frac{g_M(q_1^2)}{2M} \sigma_{\mu\nu} q_{1\nu} \right) \frac{\tau^a}{2}, \quad (3.4)$$

and the axial-vector part is analogously,

$$\hat{J}_{A,\mu}^a(q_1) = i \left( -g_A(q_1^2) \gamma_\mu \gamma_5 + i \frac{g_P(q_1^2)}{m_l} q_{1\mu} \gamma_5 \right) \frac{\tau^a}{2}. \quad (3.5)$$

Here  $a$  is the isospin index,  $M$  ( $m_l$ ) is the nucleon (lepton) mass and the 4-momentum transfer is given by  $q_{1\mu} = p'_\mu - p_\mu$ , where  $p'_\mu$  ( $p_\mu$ ) is the 4-momentum of the final (initial) nucleon.

Theoretically, all form factors entering the weak one-nucleon current were well understood and experimentally settled but one [17–19]. It was the induced pseudoscalar  $g_P$  that resisted for quite a long time. Only recently, its value has been fixed in the already mentioned MuCap experiment,  $g_P^{exp}(q^2=0.88\text{ m}_\mu^2) = 7.3 \pm 1.1$  [10, 20], which is in excellent agreement with the PCAC and chiral perturbation theory ( $\chi$ PT) prediction,  $g_P^{th}(q^2=0.88\text{ m}_\mu^2) = 8.2 \pm 0.2$  [20, 21].

#### IV. CHIRAL SYMMETRY, SOFT PIONS

The essence of the problem with the study of the above quoted nuclear reactions is that the quantum chromodynamics (QCD) is non-perturbative at low energies. The way of handling this obstacle has already been outlined 50 years ago by realizing that the description of the electro-weak interaction with a nuclear system, containing nucleons and pions, should be based on the spontaneously broken global chiral symmetry  $SU(2)_L \times SU(2)_R$  [22, 23], reflected in the QCD Lagrangian [24, 25]. This fundamental concept was realized after studying the commutation relations of the charges corresponding to the lepton currents for the system of electrons and muons with zero masses. In this case, the lepton charges satisfy the commutation relations of the group  $SU(2) \times SU(2)$  [26].

The lepton charges are defined as

$$\vec{Q}_l = \int \Psi^\dagger \frac{\vec{\tau}}{2} \Psi d^3r, \quad (4.1)$$

$$\vec{Q}_{5l} = \int \Psi^\dagger \gamma_5 \frac{\vec{\tau}}{2} \Psi d^3r, \quad (4.2)$$

where  $\vec{\tau}$  is the lepton isospin.

From Eqs. (4.1) and (4.2) one obtains

$$[Q_{il}, Q_{jl}] = i\epsilon_{ijk} Q_{kl}, \quad (4.3)$$

$$[Q_{5il}, Q_{5jl}] = i\epsilon_{ijk} Q_{kl}, \quad (4.4)$$

$$[Q_{il}, Q_{5jl}] = i\epsilon_{ijk} Q_{5kl}. \quad (4.5)$$

Gell-Mann made an assumption [27] that the universality of the weak interactions for the leptons and hadrons declares itself in that the charges connected with the hadron currents satisfy the same simultaneous commutation relations as the lepton currents.

Besides, the vector part of the weak nucleon current should satisfy the Conserved Vector Current (CVC) hypothesis, that permits the identification of the weak vector current with the isovector part of the electromagnetic current. In its turn the weak axial nucleon current should satisfy the Partially Conserved Axial Current (PCAC) hypothesis,

$$q_{1,\mu} \hat{J}_{A,\mu}^a(q_1) = i f_\pi m_\pi^2 \Delta_F^\pi(q_1^2) M_\pi^a. \quad (4.6)$$

Here  $f_\pi$  is the pion decay constant,  $m_\pi$  is the pion mass,  $\Delta_F^\pi(q_1^2) = 1/(m_\pi^2 + q_1^2)$  is the pion propagator and  $M_\pi^a$  is the matrix element of the pion production/absorption amplitude between the one-nucleon states. It is seen that the axial current is conserved in the limit of zero pion mass.

This development induced a burst of calculations and powerful low-energy theorems for the weak- and electro-production of pions on nucleon at the threshold were established [22, 23]. Since this concept is correct only for pions with  $q_1 = 0$ , the pions are called soft. Let us note that the results based on current algebras are model independent.

The spontaneously broken global chiral symmetry  $SU(2)_L \times SU(2)_R$  predicts the existence of three massless particles with the quantum numbers  $0^-$  in the ground state [28]. Since this symmetry is broken by the finite mass of the quarks, also these particles acquire finite mass, and they can be identified with pions.

Let us note that Eq.(3.1) is valid also for the nuclear processes with the electromagnetic lepton and nuclear currents.

## V. IMPULSE APPROXIMATION

Applying Eq.(3.1) in calculations of the processes in nuclei, one first supposed that the nuclear current is approximated by the sum of the one-nucleon currents (3.3). This approximation is called the Impulse Approximation (IA). Generally, this concept worked well at low energies but it was found that, in some cases, it failed to describe the data. First it happened in the case of thermal neutron capture by proton,

$$n + p \rightarrow d + \gamma, \quad (5.1)$$

that the precise experimental value of the cross section,  $\sigma^{exp} = 334.2 \pm 0.5 \text{ mb}^*$  [29], is larger by  $\approx 10 \%$  than the IA cross section,  $\sigma^{IA} = 302.5 \pm 4.0 \text{ mb}$  [30].

This reaction is triggered by the space component of the electromagnetic isovector current, which is of the order  $\sim \mathcal{O}(1/M)$ .

## VI. MESON EXCHANGE CURRENTS

Here for the first time, meson exchange currents (MECs) rescued the situation. The pion production amplitude, evaluated in the soft pion limit, provided MECs (see Fig.1c and Fig.1d) that removed about 70 % of the discrepancy [31–35].

Possible presence of the two-nucleon currents follows also from the potential description of the two-nucleon system that we consider for the sake of simplicity. In this case, the nuclear

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\* 1 barn (b) =  $10^{-24} \text{ cm}^2$

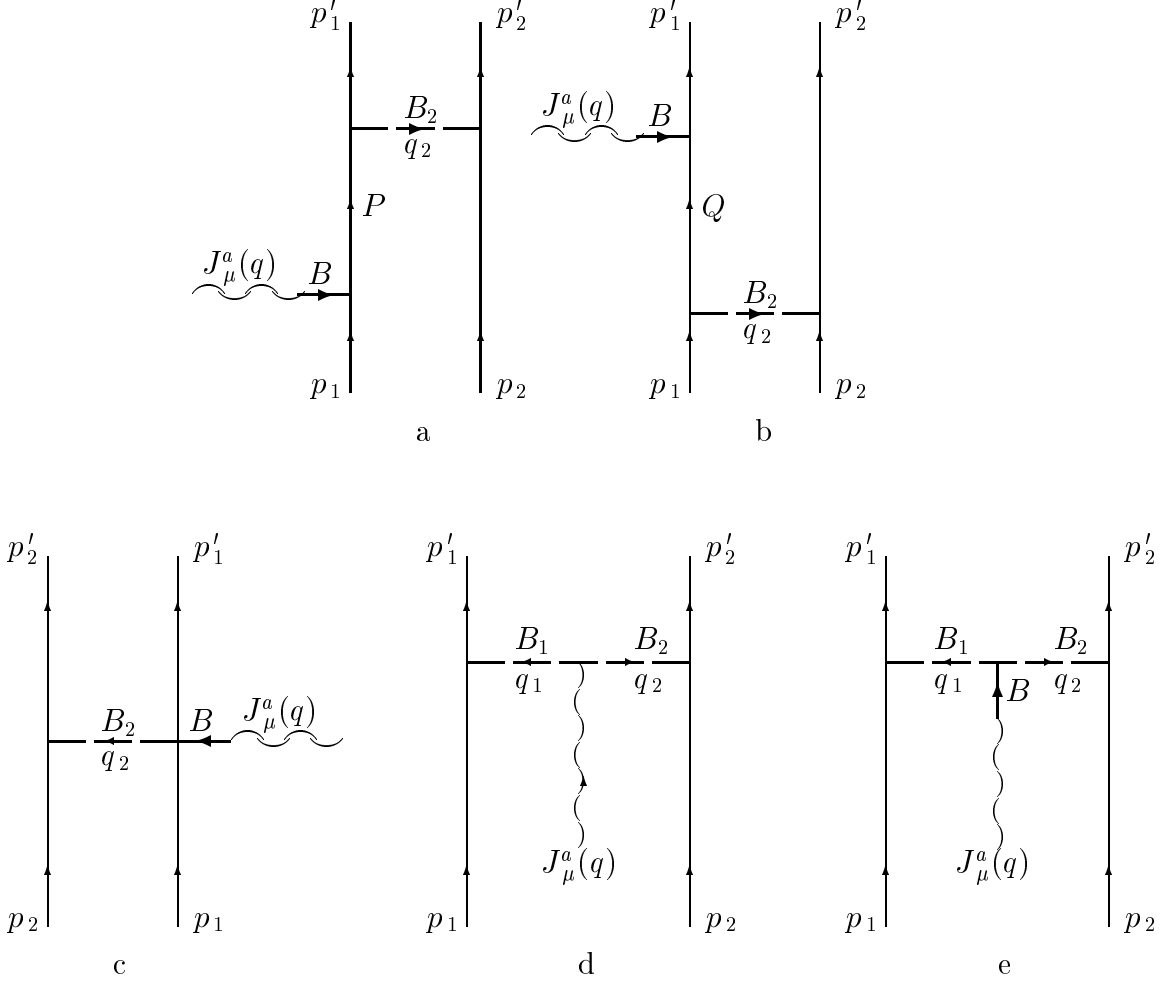


FIG. 1: The possible structure of the two-nucleon current operators;

Hamiltonian is

$$H = T + V, \quad (6.1)$$

where  $T$  is the kinetic energy and  $V$  is the nuclear potential. For the electromagnetic current,  $J_\mu(q)$ , the current conservation reads,

$$\vec{q} \cdot \vec{J}(\vec{q}) = [H, \rho(\vec{q})]. \quad (6.2)$$

Writing the current as the sum of the one-nucleon and two-nucleon parts,

$$J_\mu(q) = \sum_1^2 J_\mu(1, i, q_i) + J_\mu(2, q), \quad (6.3)$$

one gets

$$\vec{q}_i \cdot \vec{J}(1, i, \vec{q}_i) = [T_i, \rho(1, i, \vec{q}_i)], \quad i = 1, 2, \quad (6.4)$$

$$\vec{q} \cdot \vec{J}(2, \vec{q}) = [T_1 + T_2, \rho(2, \vec{q})] + ([V, \rho(1, 1, \vec{q}_1)] + (1 \leftrightarrow 2)), \quad (6.5)$$

where  $\vec{q} = \vec{q}_1 + \vec{q}_2$ .

It is seen from Eq. (6.5) that this equation cannot be fulfilled if the MECs are absent.

## VII. CHIRAL LAGRANGIANS, HARD PIONS

As we have already mentioned, the approach of soft pions is valid at the threshold energies. Next we discuss the concept of chiral Lagrangians allowing to go beyond this restriction. The approach of chiral Lagrangians is based on an assumption that any Lagrange theory satisfying the requirements

1. The constructed currents satisfy the fundamental commutation relations known from the soft-pion approach.
2. In the limit of zero pion mass the weak axial current is exactly conserved, should reproduce the results of the soft-pion technique [23]. This is possible thanks to the fact that the correct results are obtained already at the level of trees (no loops). The chiral Lagrangians of the pion-nucleon system reflect the global chiral symmetry. Standardly, they are constructed in non-linear realization of the chiral symmetry [36].

One can also consider the Lagrangians reflecting the local chiral symmetry [37, 38]. This step allowed to extend the nuclear system by the  $\rho$ - and  $a_1$  mesons, as compensating Yang-Mills fields. However, the problem was that the heavy meson masses violated the symmetry. Later on, a concept of hidden local symmetry allowed one to avoid this conceptual difficulty [39, 40]. The nonlinear hard pion chiral Lagrangians [38, 41, 42] served then as starting point for constructing the one-boson exchange currents in the tree approximation. We shall call this concept as the Tree Approximation Approach (TAA).

This approach was also applied to construct the one-boson exchange potentials [43–47]. Besides the potentials of this sort, many phenomenological potentials of various quality were also constructed [48–52]. The precise second generation potentials Nijmegen I [46], CD-Bonn [47] and AV18 [52] have  $\chi^2 \sim 1$ .

Let us note that in order to make realistic calculations, both the one-boson MECs and potentials should be supplied with the strong form factors by hands. Fortunately, this can be done in such a way that the CVC and PCAC hypotheses are still valid. Consequently, one can describe the MECs effect consistently: both the MECs and nuclear potentials are obtained within the same approach.

The reactions (1.17)-(1.18) and (2.3)-(2.5) were studied within TAA in Refs. [53–60]. These reactions are triggered by the space component of the weak current. Its one-nucleon weak axial part is of the order  $\sim \mathcal{O}(1)$ , whereas the space component of the weak axial MECs is  $\sim \mathcal{O}(1/M^2)$ . This fact makes the calculations of the weak axial MECs effects difficult.

Here we present for the reaction  ${}^2\text{H}(\mu^-, \nu_\mu)\text{nn}$  the results, calculated with the first generation potentials, for the doublet capture rate  $\Lambda_{1/2} = 416 \pm 7 \text{ s}^{-1}$  [58] and  $\Lambda_{1/2} = 397.8 - 399.6 \text{ s}^{-1}$  [59]. The total effect of the weak MECs was estimated as  $\sim 30.6 - 33.1 \text{ s}^{-1}$  [59]. Recent calculations with the precise potential Nijmegen I provided  $\Lambda_{1/2} = 416 \pm 6 \text{ s}^{-1}$  [60]. Analogous calculations for the reaction  ${}^3\text{He}(\mu^-, \nu_\mu){}^3\text{H}$  provided for the statistical rate  $\Lambda_0 = 1502 \pm 32 \text{ s}^{-1}$  [57] and  $\Lambda_0 = 1484 \pm 8 \text{ s}^{-1}$  in a purely phenomenological approach [61].

Generally, within this approach one can describe well the nuclear phenomena, triggered by the electro-weak interaction, up to energies  $\sim 1 \text{ GeV}$  [33–35, 62–68]. In Fig. 2, we present the double differential cross section for the reaction of the backward deuteron electro-disintegration [42],

$$e + d \rightarrow e' + n + p, \quad (7.1)$$

where the energetic electrons are scattered backwards, closely to 180 degrees. The detailed analysis of the contributions of particular mesons to the cross section has been made in Refs. [69, 70].

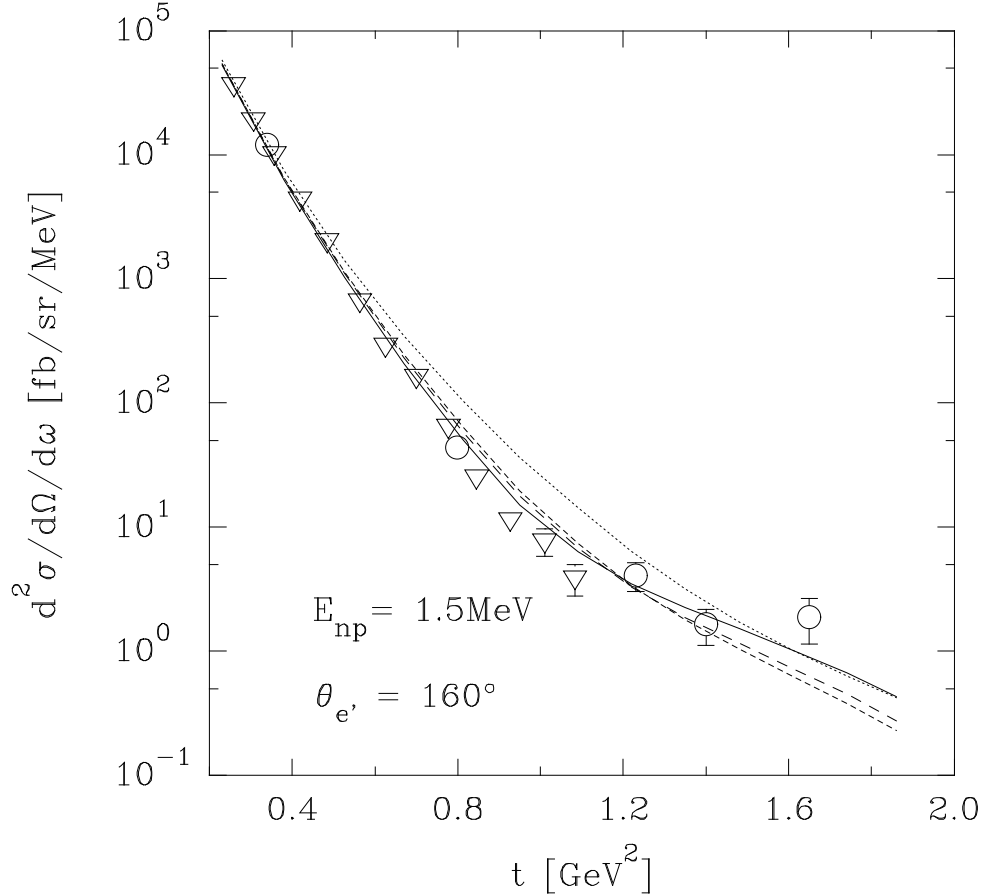


FIG. 2: The double differential cross section for the process (7.1).

From 1980's, in parallel with the above discussed schemes of the calculations in nuclear physics, new approach was worked out, providing more general framework for systematic construction of many body currents and potentials. The price for this was explicit absence of all degrees of freedom in the Lagrangian but nucleons and pions and applicability to nuclear phenomena only in the low energy region.

### VIII. EFFECTIVE FIELD THEORY

The fundamental step allowing to go beyond the tree approximation was made by Weinberg in 1979 [71]. In this work, Weinberg formulated an effective field theory (EFT): if one writes down the most general possible Lagrangian, including all terms consistent with assumed symmetry principles, and then calculates matrix elements with this Lagrangian to any given order of perturbation theory, the result will be the most general possible S-matrix consistent with perturbative unitarity, analyticity, cluster decomposition, and the assumed symmetry properties.

So in this way, one can construct dynamical theory, not limited to the tree approximation. At low energies, the effective degrees of freedom in nuclear physics are pions and nucleons, rather than quarks and gluons. With the heavy mesons and nucleon resonances integrated



out and the spontaneously broken global chiral symmetry as assumed symmetry, one obtains the  $\chi$ PT of the nucleon-pion system. The resulting effective Lagrangian is given by a string of terms, dictated by the chiral symmetry, with increasing chiral dimension [72],

$$\mathcal{L}_{\pi N} = \mathcal{L}_{\pi N}^{(1)} + \mathcal{L}_{\pi N}^{(2)} + \mathcal{L}_{\pi N}^{(3)} + \mathcal{L}_{\pi N}^{(4)} + \dots \quad (8.1)$$

This is the QCD Lagrangian of the  $\pi N$  system at low energies. The Lagrangian of dimension one is simple

$$\mathcal{L}_{\pi N}^{(1)} = \bar{\Psi}(D_\mu \gamma_\mu - M + i\frac{g_A}{2} u_\mu \gamma_\mu \gamma_5) \Psi, \quad (8.2)$$

where  $D_\mu$  and  $u_\mu$  depend non-linearly on the pion field and external interactions, and  $\gamma$ s are the Dirac matrices. Besides,  $g_A$  is the weak interaction constant.

At the second order, seven independent terms appear, at the third order, one has 23 independent terms, and at the dimension four, there are 118 independent operators [72]. Each term is multiplied by a (low energy) constant (LEC) that is fixed either in the process of the elimination of heavier resonances, or by the data.

Weinberg also established counting rules [73, 74] allowing one to classify the importance of contribution of various diagrams into perturbative expansion of the S-matrix in positive powers  $\nu$  of  $Q/\Lambda_\chi$ , where  $Q$  is a quantity characterizing the hadron system (momentum, energy or the pion mass) which is small in comparison with the heavy scale  $\Lambda_\chi \sim 1$  GeV.

As we have already noted above, in order to calculate reliably the capture rates and cross sections of reactions one needs to know accurately the nuclear wave functions (potentials) and the current operators.

### A. Space component of the weak axial MECs

The weak axial currents were constructed within the  $\chi$ PT in Refs. [75, 76]. At the leading ( $\nu=0$ )- (LO) and the next-to-leading ( $\nu=1$ ) (NLO) orders, the weak nuclear current consists of the well-known single nucleon terms. The space component of the weak axial MECs, which is of the main interest here because it enters in calculations of observables in all weak reactions mentioned above, appears at N<sup>3</sup>LO ( $\nu = 3$ ). We already know that the single nucleon current is known precisely. On the other hand, in the space component of the weak axial MECs appears one LEC, called  $\hat{d}^R$ . This parameter manifests itself in the two-nucleon contact vertex with the weak axial current (see Fig. 3). Besides, it is present also in a contact term part of the three-nucleon force constructed within the  $\chi$ PT. It follows that if this LEC will be fixed in one of the few-nucleon processes feasible in the laboratory, one can make model-independent predictions for other weak processes triggered by this component of the weak axial current.

### B. The hybrid calculations

Up to present only few calculations fulfil the requirement of consistency and so called 'hybrid' approach is applied instead: the current operator is constructed within the  $\chi$ PT as outlined above, but the wave functions are generated either from the one-boson-exchange- or phenomenological potentials of the TAA approach. Besides, in calculating the observables for the three-nucleon processes, also the three-nucleon forces Tucson-Melbourn[77] and Urbana IX [78] were used.

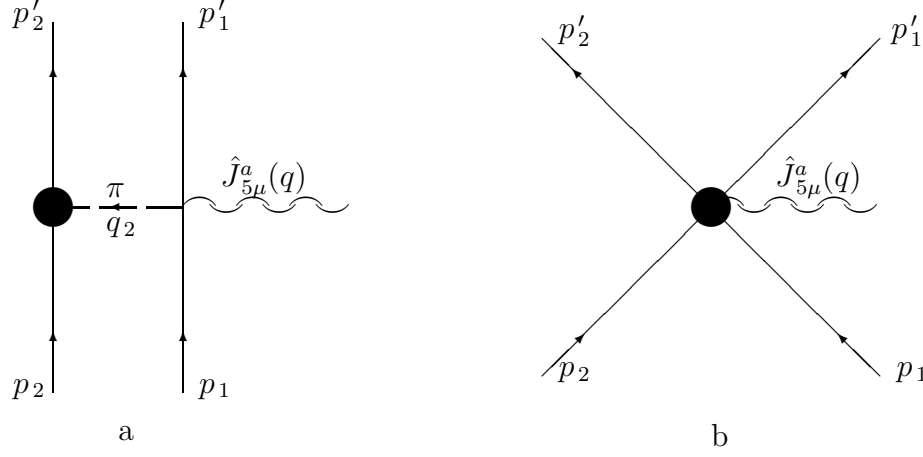


FIG. 3: The general structure of the two-nucleon weak axial operators; a– the long-range operator, b– the short-range operator.

So far almost all calculations aiming to study the weak interaction in few-nucleon systems profited from the precise knowledge of the half-life of the triton to extract the LEC  $\hat{d}^R$ . In this way, in Ref. [79], this constant was extracted from the Gamow-Teller (GT) matrix element for the reaction (2.4), and then the spectroscopic factors<sup>†</sup>  $S_{pp}(0)=3.94 \times (1 \pm 0.004) \times 10^{-25}$  MeV b and  $S_{hep}(0)=(8.6 \pm 1.3) \times 10^{-30}$  keV b were calculated for reactions of the proton-proton fusion (1.12) and the hep reaction (1.14), respectively.

Using the same values of  $\hat{d}^R$ , in Ref. [80] the doublet capture rate,  $\Lambda_{1/2}=386 \text{ s}^{-1}$ , for the reaction  ${}^2\text{H}(\mu^-, \nu_\mu)\text{nn}$  was obtained and in Ref. [81] the cross sections of the  $\nu\text{d}$  reactions, (1.17) and (1.18), were calculated. Similarly, in Ref. [82], the capture rate  $\Lambda_0 = 1499 \pm 16 \text{ s}^{-1}$  for the reaction  ${}^3\text{He}(\mu^-, \nu_\mu){}^3\text{H}$  was gained. In Ref. [83], Marcucci and Piarulli used AV18 NN potential to generate the nuclear wave functions for the process  ${}^2\text{H}(\mu^-, \nu_\mu)\text{nn}$  and AV18 + Urbana IX NNN potentials for the reaction  ${}^3\text{He}(\mu^-, \nu_\mu){}^3\text{H}$ . The weak current was taken from the  $\chi\text{PT}$  approach with the potential current of Ref. [84] added. The calculations resulted in  $\Lambda_{1/2} = 393.1 \pm 8 \text{ s}^{-1}$  and  $\Lambda_0 = 1488 \pm 9 \text{ s}^{-1}$ .

The problem with these calculations is that in the three-nucleon systems, the consistent calculations require to know two LECs,  $c_D$  and  $c_E$  [85]. The constant  $c_D$  is related to the constant  $\hat{d}^R$  as

$$\hat{d}^R = \frac{M_N}{\Lambda_\chi g_A} c_D + \frac{1}{3} M_N (\hat{c}_3 + 2\hat{c}_4) + \frac{1}{6}. \quad (8.3)$$

Here the LECs  $\hat{c}_3$  and  $\hat{c}_4$ , together with  $\hat{c}_1$ ,  $\hat{c}_2$  and  $c_6$ , are obtained either from the  $\pi\text{N}$ - [86] or NN scattering [87], or in the process of elimination of higher resonances from the general  $\chi\text{PT}$  Lagrangian [88].

In the hybrid calculations of the process  ${}^3\text{He}(\mu^-, \nu_\mu){}^3\text{H}$  mentioned above, it is not possible to fix  $c_D$  and  $c_E$  simultaneously, because  $c_E$  does not enter them. Besides, the constant  $c_D$  ( $\hat{d}^R$ ) enters not only the weak axial MECs, but also the contact and one-pion exchange part of the three-nucleon force constructed within the  $\chi\text{PT}$ , whereas the constant  $c_E$  enters the

<sup>†</sup> The spectroscopic factor is defined as  $S(E) = \sigma(E) E e^{2\pi\eta}$ , where  $E$  is the energy,  $\sigma$  is the cross section, and  $\eta$  is the Sommerfeld parameter describing the barrier penetrability.

three-nucleon contact term [89, 90].

Let us note also the work [60], where the capture rate  $\Lambda_{1/2} = 416 \pm 6 \text{ s}^{-1}$  for the reaction  ${}^2\text{H}(\mu^-, \nu_\mu)\text{nn}$  was calculated within the TAA in various current models and from this rate, the model dependence of the value of  $\hat{d}^R$  in hybrid calculations was studied.

### C. Chiral potentials

The early theory of nuclear force also started from the nucleon-pion system in 1950s but failed to describe reasonably well the empirical data. As we have already discussed, only later the chiral symmetry was recognized as proper symmetry of the Nature. In 1960s, the heavy mesons were discovered and used to construct quite successful models of nuclear force. The price for it was the necessity to introduce phenomenological strong form factors into the one-boson-exchange potentials and to deal with a not well known scalar-isoscalar  $\sigma$  meson. With the advent of QCD, the attempts to construct the nuclear forces from the QCD-inspired quark models appeared. But only within the  $\chi$ PT it was possible to derive consistently the nuclear force between the  $N$  nucleons from the same Lagrangian and precise two-nucleon potentials have been constructed up to  $N^3\text{LO}$ : the Entem-Machleidt (EM) [87] and Epelbaum-Gloeckle-Meißner [91] potentials. In both cases, the entering parameters are standardly extracted from the fit to the nucleon-nucleon scattering data and the deuteron properties, or some of them are adopted from the analysis of the  $\pi N$  scattering [91].

The three-nucleon force has been derived within the  $\chi$ PT, but only up to  $N^2\text{LO}$  [89, 90] so far. As we already know, this force contains two constants,  $c_D$  and  $c_E$ , to be determined. The first consistent extraction of the constants  $c_D$  and  $c_E$  from the three-nucleon system has recently been made by Gazit, Qualioni and Navrátil in Ref. [85], where these constants are constrained by simultaneous calculations of the binding energies of the three-nucleon systems and of the triton  $\beta$  decay. The nuclear wave functions are generated in accurate *ab initio* calculations using both the two-nucleon EM [87] and three-nucleon  $N^2\text{LO}$  force [89, 90], whereas the process (2.4) is calculated with the weak axial MECs derived from the same  $\chi$ PT Lagrangian [82] as the nuclear forces. The resulting values of the two constants are restricted in the intervals,  $-0.3 \leq c_D \leq -0.1$ ,  $-0.220 \leq c_E \leq -0.189$ .

The reactions of muon capture in deuterium and in  ${}^3\text{He}$  have very recently been calculated by the Pisa group (MEAL) [93] in the TA- and hybrid approaches and within the  $\chi$ PT as well. The resulting values of the capture rates are,  $\Lambda_{1/2} = (389.7 - 394.3) \text{ s}^{-1}$  and  $\Lambda_0 = (1471 - 1497) \text{ s}^{-1}$ .

In order to compare our results with those of MEAL, we choose the last row of TABLE VI [93] in which we divide the items by 1.024 [20]. This number represents the inner radiative corrections taken into account by MEAL. In our analogous  $\chi$ PT calculations [94] with the same  $N^3\text{LO}$  potential [87], we take the value of  $c_D = -0.2$  [85] and  $\Lambda = 500 \text{ MeV}$ . Besides, using the values of the LECs (in unit  $\text{GeV}^{-1}$ ),  $c_1 = -0.81$ ,  $c_2 = 2.80$ ,  $c_3 = -3.20$  and  $c_4 = 5.40$  [87], we obtain from Eq. (8.3)  $\hat{d}^R = 2.400$ . Let us note that this value of  $\hat{d}^R$  differs considerably from  $\hat{d}^R = 1.00(9)$ , presented in the last row of TABLE V [93].

The results of calculations are given in Table I. As it is seen, the main difference stems from the  ${}^1\text{S}_0$  and  ${}^3\text{P}_1$  waves. Also the contributions for the  ${}^1\text{D}_2$  wave differ, but they are small. Since the calculated MECs effect,  $\Delta\text{MEC} = 15.4 \text{ s}^{-1}$  [94], is similar to the one obtained in [60, 80], we conclude that the difference comes from the IA calculations in the channels  ${}^1\text{S}_0$  and  ${}^3\text{P}_1$ .

TABLE I: Contributions to  $\Lambda_{1/2}$  from partial waves (in  $\text{s}^{-1}$ ). In the last column, all the contributions are summed up. RT - our calculations, MEAL - the last column of TABLE VI [93] divided by 1.024.

	$^1\text{S}_0$	$^3\text{P}_0$	$^3\text{P}_1$	$^3\text{P}_2$	$^1\text{D}_2$	$^3\text{F}_2$	total
RT	258.7	19.6	59.5	69.7	6.2	0.4	414.1
MEAL	244.6	19.4	45.3	69.8	4.3	0.9	384.3

## IX. CONCLUSIONS

We have seen that the constant  $\hat{d}^R$  ( $c_E$ ) is currently extracted from the triton beta decay rate. However, it is abundantly clear from our discussion that dealing with the complexity of the three-nucleon system can be avoided, if  $\hat{d}^R$  would be determined accurately from the muon capture in deuterium. Then one can consistently calculate other two-nucleon weak processes, such as proton-proton fusion reaction (1.12) and both reactions, (1.17) and (1.18), of solar neutrinos with the deuterons, so important for the astrophysics. Besides it turns out [92] that  $\hat{d}^R$  enters also the capture rate for the reaction  $\pi^- + d \rightarrow \gamma + 2n$ , which is the best source of information on the neutron-neutron scattering length  $a_{nn}$ .

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- [1] R.D.E. Atkinson and F.G. Houtermans, Z. Physik **54** (1929) 654.
  - [2] H.A. Bethe, Phys. Rev. **55** (1939) 434 .
  - [3] E.G. Adelberger *et al.*, Solar fusion cross sections II: the pp chain and CNO cycle, arXiv: 1004.2318.
  - [4] SNO Collaboration, Q.R. Ahmad *et al.*, Phys. Rev. Lett. **87** (2001) 071301 ; Phys. Rev. Lett. **89** (2002) 011301 ; Phys. Rev. Lett. **89** (2002) 011302 .
  - [5] SNO Collaboration, S.N. Ahmed *et al.*, Phys. Rev. Lett. **92** (2004) 181301 .
  - [6] SNO Collaboration, B. Aharmim *et al.*, Phys. Rev. **C72** (2005) 055502 ; Phys. Rev. **C75** (2007) 045502 .
  - [7] KamLAND Collaboration, S. Abe *et al.*, Phys. Rev. Lett. **100** (2008) 221803 .
  - [8] R. Martin, Results from the Neutral Current Detector phase of the Sudbury Neutrino Observatory, ArXiv: 0905.4907.
  - [9] A.P. Serebrov and A.K. Fomin, A new world average value for the neutron lifetime, ArXiv: 1005.4312.
  - [10] MuCap Collaboration, V.A. Andreev *et al.*, Phys. Rev. Lett. **99** (2007) 032002 ;
  - [11] Yu.A. Akulov and B.A. Mamyurin, Phys. Lett. **B610** (2005) 45 .
  - [12] A.A. Vorobyov *et al.* Hyperfine Interactions **101/102** (1996) 413 .
  - [13] P. Ackerbauer *et al.* Phys. Lett. **B417** (1998) 224 .
  - [14] J. Martino, Czech. J. Phys. **B36** (1986) 368 .
  - [15] M. Cargnelli, PhD thesis, Technical University of Vienna, 1987; W.H. Breunlich, M. Cargnelli, H. Fuhrmann, P. Kammel, J. Marton, J. Werner, J. Zmeskal and C. Petitjean, in Workshop on fundamental  $\mu$ -physics, Los Alamos, 1986, LA10714C.
  - [16] MuSun Collaboration, Muon Capture on the Deuteron, <http://www.npl.uiuc.edu/exp/musun>.

- [17] C. Amsler *et al.*, Phys. Lett. **B667** (2008) 1 .
- [18] P. Kammel, Muon capture and muon lifetime, talk at the workshop Fundamental Symmetries and Weak Interactions, Institute for Nuclear Theory, University of Washington, Sept.30–Dec.6, 2002.
- [19] A. Liesenfeld *et al.*: Phys. Lett. **B468** (1999) 20 .
- [20] A. Czarnecki, W.J. Marciano and A. Sirlin, Phys. Rev. Lett. **99** (2007) 032003 .
- [21] S.L. Adler and Y. Dothan, Phys. Rev. **151** (1966) 1267 .
- [22] S.L. Adler and R. Dashen, Current Algebras (W.A. Benjamin, New York, 1968).
- [23] V. De Alfaro, S. Fubini, G. Furlan and C. Rossetti, Currents in Hadron Physics (North–Holland/American Elsevier, Amsterdam–London/New York, 1973).
- [24] J.F. Donoghue, E. Golowich and B.R. Holstein, Dynamics of the Standard Model (Cambridge University Press, Cambridge, 1992).
- [25] S. Weinberg, The Quantum Theory of Fields, v. II (Cambridge University Press, Cambridge, 1996).
- [26] J. Bernstein, Elementary particles and their currents (W.H. Freeman and Company, San Francisco and London, 1968).
- [27] M. Gell-Mann, Physics **1** (1964) 63 .
- [28] J. Goldstone, Nuovo Cim. **9** (1961) 154 .
- [29] A.E. Cox, S.A.R. Wynchank and C.H. Collie, Nucl. Phys. **74** (1965) 507 .
- [30] H.P. Noyes, Nucl. Phys. **74** (1965) 508 .
- [31] M. Chemtob and M. Rho, Nucl. Phys. **A163** (1971) 1 .
- [32] D.O. Riska and G.E. Brown, Phys. Lett. **B38** (1972) 193 .
- [33] J.-F. Mathiot, Phys. Rep. **173** (1989) 63 .
- [34] D.O. Riska, Phys. Rep. **181** (1989) 207 .
- [35] T. Ericson and W. Weise, Pions and Nuclei
- [36] S. Weinberg, Phys. Rev. **166** (1968) 1568 ; S. Coleman, J. Wess and B. Zumino, Phys. Rev. **177** (1969) 2239 ; C.G. Callan, S. Coleman, J. Wess and B. Zumino, Phys. Rev. **177** (1969) 2247 .
- [37] J. Schwinger, Phys. Lett. **B24** (1967) 473 .
- [38] V.I. Ogievetsky and B.M. Zupnik, Nucl. Phys. **B24** (1970) 612 .
- [39] U.-G. Meissner, Phys. Rep. **161** (1988) 213 .
- [40] M. Bando, T. Kugo and K. Yamawaki, Phys. Rep. **164** (1988) 217 .
- [41] E. Ivanov and E. Truhlík, Nucl. Phys. **A316** (1979) 437 .
- [42] J. Smejkal, E. Truhlík and H. Göller, Nucl. Phys. **A624** (1997) 655 .
- [43] R. Bryan and B.L. Scott, Phys. Rev. **177** (1969) 1435 .
- [44] R. Machleidt, K. Holinde and Ch. Elster, Phys. Rep. **149** (1987) 1 .
- [45] R. Machleidt, Adv. Nucl. Phys. **19** (1989) 189 .
- [46] V.G.J. Stoks, R.A.M. Klomp, C.P.F. Terheggen and J.J. de Swart, Phys. Rev. **C49** (1994) 2950 .
- [47] R. Machleidt, Phys. Rev. **C63** (2001) 024001 .
- [48] T. Hamada and I.D. Johnston, Nucl. Phys. **34** (1962) 382 .
- [49] R.V. Reid, Jr., Ann. Phys. (N.Y.) **50** (1968) 411 .
- [50] R. De Tourreil, B. Rouben and D.W.L. Sprung, Nucl. Phys. **A242** (1975) 445 .
- [51] M. Lacombe *et al.*, Phys. Rev. **C21** (1980) 861 .
- [52] R.B. Wiringa, V.G.J. Stoks and R. Schiavilla, Phys. Rev. **C51** (1995) 38 .
- [53] S. Nakamura, T. Sato, S. Ando, T.-S. Park, F. Myhrer, V. Gudkov and K. Kubodera, Nucl.

- Phys. **A707** (2002) 561 .
- [54] B. Mosconi, P. Ricci and E. Truhlík, Nucl. Phys. **A772** (2006) 81 .
  - [55] B. Mosconi, P. Ricci and E. Truhlík and P. Vogel, Phys. Rev. **C75** (2007) 044610 .
  - [56] S. Ciechanowicz and E. Truhlík, Nucl. Phys. **A414** (1984) 508 .
  - [57] J. Congleton and E. Truhlík, Phys. Rev. **C53** (1996) 956 .
  - [58] J. Adam Jr. E. Truhlík, S. Ciechanowicz and K.M. Schmitt, Nucl. Phys. **A507** (1990) 675 .
  - [59] N. Tataru, Y. Kohyama and K. Kubodera, Phys. Rev. **C42** (1990) 1694 .
  - [60] P. Ricci, E. Truhlík, B. Mosconi and J. Smejkal, Nucl. Phys. **A837** (2010) 110 .
  - [61] L.E. Marcucci, R. Schiavilla, S. Rosati, A. Kievsky and M. Viviani, Phys. Rev. **C66** (2002) 054003 .
  - [62] E.A. Ivanov and E. Truhlík, Sov. J. Part. Nucl. **12** (1980) 198 .
  - [63] M. Kirchbach and E. Truhlík, Sov. J. Part. Nucl. **17** (1986) 93 .
  - [64] I.S. Towner, Phys. Rep. **155** (1987) 263 . (Clarendon Press, Oxford, 1988).
  - [65] J. Carlson and R. Schiavilla, Rev. Mod. Phys. **70** (1998) 743 .
  - [66] D.F. Measday, Phys. Rep. **354** (2001) 243 .
  - [67] I. Sick, Prog. Part. Nucl. Phys. **47** (2001) 245 .
  - [68] R. Gilman and F. Gross, J. Phys. **G28** (2002) R37 .
  - [69] E. Truhlík and J. Adam, Jr. Nucl. Phys. **A492** (1989) 529 .
  - [70] E. Truhlík and K.-M. Schmitt, Few-Body Systems **11** (1992) 155 .
  - [71] S. Weinberg, Physica **96A** (1979) 327.
  - [72] N. Fettes, U.-G. Meißner, M. Mojzis and S. Steiniger, Ann. Phys. (N.Y.) **283** (2000) 273  
[Erratum-ibid. **288** (2001) 249].
  - [73] S. Weinberg, Phys. Lett. **B251** (1990) 292 .
  - [74] S. Weinberg, Nucl. Phys. **B363** (1991) 3 .
  - [75] T.-S. Park, K. Kubodera, D.-P. Min and M. Rho, Astrophys. J. **507** (1998) 443 .
  - [76] D. Gazit, Ph.D. Thesis, The Hebrew University of Jerusalem, 2007, arXiv: 0807.0216; Phys. Lett. **B666** (2008) 472 .
  - [77] S.A. Coon, M.D. Scadron, P.C. McNamee, B.R. Barret, D.W.E Blatt and B.H.J. McKellar, Nucl. Phys. **A317** (1979) 242 .
  - [78] B.S. Pudliner, V.R. Pandharipande, J. Carlson and R.B. Wiringa, Phys. Rev. Lett. **74** (1995) 4396 .
  - [79] T.-S. Park, L.E. Marcucci, R. Schiavilla, M. Viviani, A. Kievsky, S. Rosati, K. Kubodera, D.-P. Min and M. Rho, Phys. Rev. **C67** (2003) 055206 .
  - [80] S. Ando, T.-S. Park, K. Kubodera and F. Myhrer, Phys. Lett. **B533** (2002) 25 .
  - [81] S. Ando, Y.H. Song, T.-S. Park, H.W. Fearing and K. Kubodera, Phys. Lett. **B555** (2003) 49 .
  - [82] D. Gazit, Phys. Lett. **B666** (2008) 472 .
  - [83] L.E. Marcucci and M. Piarulli, Muon capture on light nuclei, arXiv: 1009.4016 [nucl-th].
  - [84] P. Ricci, B. Mosconi and E. Truhlík, Eur. Phys. J. **A25** (2005) 283 .
  - [85] D. Gazit, S. Quaglioni and P. Navrátil, Phys. Rev. Lett. **103** (2009) 102502 .
  - [86] V. Bernard, Prog. Part. Nucl. Phys. **60** (2008) 82 .
  - [87] D.R. Entem and R. Machleidt, Phys. Rev. **C68** (2003) 041001 (R) .
  - [88] V. Bernard, N. Kaiser and U.G. Meissner, Int. J. Mod. Phys. **E4** (1995) 193 .
  - [89] U. van Kolck, Phys. Rev. **C49** (1994) 2932 .
  - [90] E. Epelbaum, A. Nogga, W. Gloeckle, H. Kamada, U.-G. Meißner and H. Witala, Phys. Rev. **C66** (2002) 064001 .

- [91] E.Epelbaum, W. Gloeckle and U.-G. Meißner, Nucl. Phys. **A747** (2005) 362 .
- [92] A. Gardestig and D.R. Phillips, Phys. Rev. **C73** (2006) 014002 .
- [93] L.E. Marcucci, M. Piarulli, M. Viviani, L. Girlandia, A. Kievsky, S. Rosati and R. Schiavilla, Muon capture on deuteron and  $^3\text{He}$ , arXiv:1008.1172 [nucl-ph].
- [94] P. Ricci and E. Truhlík, to be published.